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Enhancing Risk Management Practices: Insights from VAR Forecasting in Financial Markets

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Abstract
This paper provides valuable insights into the challenges and opportunities associated with forecasting VAR in financial markets. By analyzing the strengths and limitations of different modeling techniques, the study offers guidance on selecting the most appropriate approach based on the characteristics of the underlying data and the specific requirements of risk management practices. Moreover, the research contributes to the broader literature on risk management by addressing the ongoing debate surrounding the adequacy of traditional VAR models in capturing extreme events and tail risk. By demonstrating the effectiveness of EVT-based approaches, the paper underscores the importance of adopting more robust and flexible methodologies to account for the inherent uncertainty and volatility in financial markets. Furthermore, the study emphasizes the need for continuous refinement and improvement in VAR modeling techniques to adapt to evolving market conditions and regulatory requirements. As financial markets become increasingly interconnected and complex, the ability to accurately quantify and manage risk becomes paramount for investors, financial institutions, and policymakers alike. In this context, the findings of the paper not only contribute to advancing our understanding of VAR forecasting but also provide practical implications for enhancing risk management practices in the financial industry. By incorporating insights from EVT and other advanced modeling techniques, stakeholders can better navigate the challenges posed by market volatility and uncertainty, thereby mitigating potential losses and safeguarding financial stability. Overall, the paper serves as a valuable resource for researchers, practitioners, and policymakers seeking to enhance their understanding of VAR modeling and improve the effectiveness of risk management strategies in the dynamic and ever-changing landscape of financial markets.
Keywords: VAR Forecasting, Risk Management, Financial Markets, Extreme Value Theory, Modeling Techniques
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1. INTRODUCTION

In recent decades, financial markets have experienced significant losses due to unexpected market crashes. These high-profile financial disasters have prompted financial institutions, regulators, and academics to engage in extensive research to develop better measurement techniques and hedging tools against market risk. One of the most widely used risk measures in the financial industry is Value-at-Risk (VaR), which was originally proposed by J.P. Morgan in 1994. This measure has been extensively discussed and refined by various scholars, including Duffie and Pan (1997) and Engle and Manganelli (2004). Value-at-Risk (VaR) is designed to quantify the potential loss in value of a portfolio over a defined period for a given confidence interval. By providing a clear metric for potential losses, VaR helps financial institutions understand and manage their exposure to market risks. It essentially estimates the maximum potential loss that could occur with a specified probability due to market movements. The introduction of VaR marked a significant advancement in risk management practices within the financial industry. Its ability to standardize risk measurement across different types of assets and portfolios made it a fundamental tool for financial risk managers. VaR is used not only for internal risk management but also for regulatory purposes, helping institutions meet capital adequacy requirements set by financial regulators. In response to the limitations and criticisms of VaR, particularly its inability to predict extreme market events (also known as "tail risks"), researchers have developed various extensions and alternative risk measures. These include Conditional Value-at-Risk (CVaR), which provides an expected loss given that the loss has exceeded the VaR threshold, and other sophisticated models that incorporate stress testing and scenario analysis.

Value-at-Risk (VaR) is defined as the maximum potential loss on a portfolio, with a given probability, over a specific time horizon. Essentially, VaR simplifies the risk associated with a portfolio to a single number, representing the loss that could occur with a certain probability. This measure has become a cornerstone in risk management because of its ability to provide a clear and quantifiable assessment of risk. However, VaR has limitations, particularly in capturing extreme market events or "tail risks." To address these limitations, researchers have applied Extreme Value Theory (EVT) to estimate the tail distribution of financial time series. EVT focuses on the behavior of extreme deviations from the median of probability distributions, making it particularly useful for assessing the risk of rare but severe market

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events. The application of EVT in financial risk management has led to a significant body of empirical research. Studies by Bali (2003, 2007), Danielsson and de Vries (1997a, 1997b, 1997c), Danielsson et al. (1998), Embrechts et al. (1998, 1999), Longin (1997a, 1997b), and McNeil (1997, 1998) have explored various aspects of tail risk and its implications for financial markets. These researchers have contributed to a deeper understanding of how extreme market movements can be modeled and managed. Bali's research, for instance, investigates the application of EVT to financial returns and provides methodologies for better estimating the tail risk. Danielsson and de Vries have explored the use of EVT in financial econometrics, emphasizing its importance in understanding and predicting extreme market movements. Embrechts and his colleagues have contributed significantly to the theoretical underpinnings of EVT in finance, helping to refine the models used to estimate risk in the tails of the distribution.

Longin's work focuses on the historical frequency of extreme market events and how EVT can be applied to predict the likelihood of such events in the future. McNeil's contributions have been pivotal in developing practical applications of EVT for risk management, providing tools and techniques for financial institutions to implement EVT-based risk measures. Focusing on extreme returns rather than the entire distribution, Extreme Value Theory (EVT) has the potential to outperform other approaches in predicting unexpected extreme changes. Studies by Dacorogna et al. (1995) and Longin (2000) highlight the advantages of EVT in capturing the tail risks that traditional models might miss. However, a significant challenge with most previous EVT-based methods for quantile estimation is that they often yield VaR estimates that do not adequately reflect the current volatility environment. These methods typically assume that the underlying market conditions remain constant, which is rarely the case in dynamic financial markets. As market volatility changes, so too does the risk profile of the portfolio, which should be reflected in the VaR estimates. To address this limitation, more sophisticated approaches have been developed that integrate current volatility levels into the VaR estimation process. One such approach involves using GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models to estimate the changing volatility over time and then applying EVT to the residuals of these models. This combination allows for more accurate and responsive risk assessments, as it captures both the tail behavior and the changing volatility of financial time series. By incorporating current volatility, these enhanced EVT-based models provide a more realistic measure of risk that aligns better with the present market conditions. This not only improves the accuracy of VaR estimates but also enhances the ability of financial institutions to manage risk effectively. As financial markets continue to evolve, ongoing research and development in this area are crucial to ensuring that risk measurement techniques remain robust and relevant. To address the limitations of traditional EVT-based methods for quantile estimation, McNeil and Frey (2000) proposed a combined approach that accounts for two key characteristics of financial return series: stochastic volatility and the fat-tailedness of the conditional returns distribution. This approach enhances the predictive accuracy of risk measures by integrating both the volatility dynamics and the extreme behavior of returns.

Numerous empirical studies have investigated the effectiveness of the EVT approach in forecasting risk across various financial markets, underscoring its importance in market risk management. For instance, Bali and Neftçi (2003) explored EVT's application in measuring market risk, highlighting its advantages over traditional models. Bystrom (2004, 2005) examined the performance of EVT in different market conditions, demonstrating its robustness in capturing extreme risks. Fernandez (2005) assessed the predictive capabilities of EVT in the context of stock markets, finding that EVT provides more accurate risk forecasts than conventional methods. Bali (2007) further analyzed the use of EVT in estimating tail risk, confirming its superiority in predicting extreme market movements. Cotter (2007) focused on the applicability of EVT in derivative markets, showing its effectiveness in managing market risk for complex financial instruments. Ghorbel and Trabelsi (2008) investigated the use of EVT in emerging markets, illustrating its utility in environments characterized by higher volatility and risk. Marimoutou, Raggad, and Trabelsi (2009) extended this analysis to various asset classes, reinforcing the broad applicability of EVT-based risk measures. Finally, Karmakar (2013) explored the integration of EVT with other risk management tools, demonstrating its potential in enhancing overall risk assessment frameworks. These studies highlight the significance of EVT in improving risk measurement and management. By focusing on the extremes and incorporating the variability in volatility, the EVT approach offers a more comprehensive and accurate assessment of market risks, making it an invaluable tool for financial institutions aiming to mitigate potential losses from extreme market events.

To the best of our knowledge, the empirical literature has not thoroughly considered the potential long-range memory in financial asset return volatilities when using EVT to quantify market risks. The existing studies have primarily focused on aspects like heteroscedasticity, fat tails, and normality in the empirical distribution of return time series. For instance, Degiannakis (2004) analyzed the forecasting performance of various risk models for estimating one-day-ahead realized volatility and daily VaR. He concluded that the Fractional Integrated APARCH (FIAPARCH) model with skewed-student-t conditionally distributed innovations is more suitable for capturing the major stylized facts of equity price behavior. Long-range memory, or long-term dependence, refers to the phenomenon where past values in a time series have a significant impact on future values over a longer horizon. This characteristic is crucial for accurately modeling financial time series, as it can influence the persistence of volatility and the occurrence of extreme events. The FIAPARCH model incorporates both fractional integration and the Asymmetric Power ARCH process, making it adept at modeling long-range dependence and capturing the asymmetry and fat tails observed in financial return distributions. By integrating skewed-student-t distributions, the model also addresses the skewness in financial returns, providing a more comprehensive framework for risk estimation. Incorporating long-range memory into the EVT framework could enhance the accuracy of risk measures such as VaR, especially under volatile market conditions. It would allow for a better understanding of the persistence of extreme risks and improve the robustness of risk management strategies.

Future research could focus on integrating long-range memory into EVT-based models, further advancing the methodologies for market risk quantification and providing financial institutions with more reliable tools for managing extreme market events.

Tang and Shieh (2006) have explored the long-memory properties of three stock index futures markets, concluding that the hyperbolic GARCH (HYGARCH) model with a skewed-student-t distribution performs better. Similar conclusions were drawn by Kang and Yoon (2007). In a related study, Mabrouk and Saadi (2012) estimated Value-at-Risk (VaR) using three long-memory models—FIGARCH, HYGARCH, and FIAPARCH—under different error distribution assumptions for seven stock market indices. They found that models accounting for asymmetries in volatility specifications and fractional integration in the volatility process outperform others in predicting VaR. Building on these findings, the focus of this paper is to explore the utility of Extreme Value Theory (EVT) in predicting extreme risks in financial markets. We employ the methodology proposed by McNeil and Frey (2000), integrating long-memory GARCH-type models to forecast financial indices' volatility and using EVT to model the tails of the distribution. Specifically, we concentrate on computing the VaR for long trading positions under different confidence levels across various financial markets. The integration of EVT with long-memory GARCH-type models aims to address the limitations of traditional risk measurement techniques, particularly in capturing the persistence and extremity of financial risks. By focusing on the tails of the distribution, EVT provides a more accurate estimation of extreme market movements, which is crucial for effective risk management. To implement this approach, we adopt several long-memory GARCH-type models, including FIGARCH, HYGARCH, and FIAPARCH, known for their ability to capture the long-range dependence and volatility clustering observed in financial return series. By combining these models with EVT, we enhance the robustness of VaR estimates, offering a comprehensive framework for assessing market risk. Our empirical analysis involves evaluating the performance of these models in forecasting volatility and computing VaR across different financial indices. We test the models under various confidence levels to ensure their reliability and effectiveness in different market conditions. The results of this study are expected to provide valuable insights into the effectiveness of incorporating long-memory and EVT in risk management practices, ultimately contributing to more resilient financial systems.

2. DATA DESCRIPTION

In this paper, we aim to estimate risk measures for financial markets using closing daily spot prices of four major market stock indices: S&P 500 (U.S.), CAC 40 (France), Nikkei 225 (Japan), and the exchange rate Euro/USD. The data covers the period from March 2003 to May 2011, totaling over 2000 observations. Continuously compounded daily returns are computed as follows:

$rt=\ln (Pt-1Pt)\times 100$

where $rtrt$ is the return in percent and $PtPt$ is the closing price of the stock index on day tt.

For each series under study, the dataset is subdivided into two subsets. The first subset is used for in-sample analysis, while the last 600 daily returns are reserved for out-of-sample analysis. This division allows us to test the models' predictive performance on unseen data, ensuring the robustness of our risk estimation methods. By focusing on these prominent indices, we cover a broad spectrum of the global financial market, allowing us to assess the applicability and effectiveness of our models in diverse economic contexts. The selected period encompasses various market conditions, including bullish, bearish, and volatile phases, providing a comprehensive dataset for our analysis. The use of continuously compounded returns is standard in financial econometrics as it offers several advantages over simple returns, including the ease of mathematical manipulation and the ability to handle multiplicative processes naturally. This approach allows for more accurate modeling of financial time series and better risk measurement. In the subsequent sections, we will apply a combination of long-memory GARCH-type models and Extreme Value Theory (EVT) to estimate Value-at-Risk (VaR) for these indices. The in-sample analysis will involve fitting the models to the first subset of the data, while the out-sample analysis will test the predictive power of these models on the last 600 observations. This comprehensive methodology aims to provide reliable risk measures that can be used by financial institutions to manage market risk effectively.

3. RESULTS AND DISCUSSIONS

Table 1 provides a comprehensive overview of summary statistics, unit root tests, stationarity tests, and long-memory tests for daily log-returns of four financial instruments: S&P 500, CAC 40, Nikkei, and Euro/USD. Panel A presents the descriptive statistics, offering insights into the distribution of daily log-returns for these instruments. The mean indicates the average daily return, with the S&P 500 having the highest mean return of 0.0169 and Nikkei the lowest at 0.0083. The maximum values show the highest daily returns observed, with Euro/USD reaching up to 16.4137. Conversely, the minimum values display the lowest daily returns, with Euro/USD experiencing the largest negative return of -12.8267. The standard deviation measures the volatility of returns, highlighting Euro/USD as the most volatile with a standard deviation of 2.5519. Skewness indicates the asymmetry of the return distribution, where the S&P 500 and Nikkei have negative skewness, suggesting a distribution with a longer tail on the left, while CAC 40 and Euro/USD have positive skewness. The kurtosis values reflect the peakedness of the distribution, and all instruments exhibit significant excess kurtosis, indicating fat tails and a higher probability of extreme values. The Jarque-Bera test results suggest non-normality in the return distributions, as evidenced by extremely high values and significant p-values (<0.0001). Finally, the Q2(10) values indicate strong autocorrelation in squared returns.

Panel B details the unit root and stationary tests, which examine whether the return series are stationary. The ADF test results show that all series are significantly stationary at the 1% level, with p-values of 0.0000. Similarly, the PP test confirms the stationarity of the series. The KPSS test, which tests the null hypothesis of stationarity, yields values below critical levels, indicating that the series are stationary. Panel C evaluates the presence of long memory in the return and squared return series. Lo's R/S test for returns indicates significant long-memory effects for the S&P 500 and CAC 40, with p-values of 0.0191 and 0.0388, respectively. However, Nikkei and Euro/USD do not show significant long-memory in returns. The test results for squared returns reveal significant long-memory effects across all instruments, suggesting the presence of volatility clustering. Furthermore, the test for absolute returns indicates significant long-memory effects in all instruments, reinforcing the observation of volatility clustering. These findings suggest that while the daily returns of these instruments are generally stationary, they exhibit significant long-memory effects in both squared and absolute returns, a common characteristic of financial time series data due to volatility clustering.

Table 1: summary statistics, unit root, stationarity tests and long-memory tests for daily log-returns

panel A: descriptive statistics	S&P500		CAC40		NIKKEI		Euro/USD	
Mean	0,0169		0,0140		0,0083		0,0608	
Maximum	10,9572		10,5946		13,2346		16,4137	
Minimum	-9,4695		-9,4715		-12,1110		-12,8267	
S,D	1,3203		1,4098		1.6232110		2,5519	
Skweness	-0,2801***		0,1020*		-0,5913***		0,2077***	
	(0,0000)		(0,0596)		(0,0000)		(0,0001)	
kurtosis (excess)	11,7240***		8,6494***		8,8499***		2,8472***	
	(0,0000)		(0,0000)		(0,0000)		(0,0000)	
J-B test	11566,5616		6396,8432		6510,4058		719,9672	
	(0,0000)		(0,0000)		(0,0000)		(0,0000)	
$Q^2(10)$	1945,0800		1121,4900		1901,8400		287,5670	
	(0,0000)		(0,0000)		(0,0000)		(0,0000)	
panel B: unit root and stationary tests								
ADF test	-27,1332***	(0,0000)	-29,4181***	(0,0000)	-26,7350***	(0,0000)	-26,6965***	(0,0000)
PP test	-50,6839***	(0,0000)	-49,0775***	(0,0000)	-44,5953***	(0,0000)	-46,0758***	(0,0000)
KPSS test	0,1315	(0.463)	0,2383	(0.463)	0,3138	(0.463)	0,0552	(0.463)
Panel C: long-memory test statistics								
return $\left(r_t\right)$								
Lo's R/S test	1,6190**	(0.0191)	1,5610**	(0.0388)	1,3983	(0.4079)	1,1730	(0.6524)
Squared return $\left(r_t^2\right)$								
Lo's R/S test	5,1586***	(0,0000)	4,5799***	(0,0000)	3,85971***	(0,0000)	4,4222***	(0,0000)
Absolute return $\left\|r_t\right\|$								
Lo's R/S test	6,9368***	(0,0000)	6,298***	(0,0000)	4,9745***	(0,0000)	5,3945***	(0,0000)

In Panel A of Table 2, the estimation results for the FIGARCH model are presented for four financial instruments: S&P500, CAC40, NIKKEI250, and EURO/USD. The GARCH coefficient estimates, denoted by "GARCH," represent the persistence of volatility shocks in the model. All coefficients are statistically significant at the 1% level, indicating a significant persistence of volatility shocks in all series. The ARCH coefficients, denoted by "ARCH," capture the effect of past squared residuals on current conditional variance. While the ARCH coefficient for S&P500 is not statistically significant, indicating no significant impact of past squared residuals, the ARCH coefficients for CAC40, NIKKEI250, and EURO/USD are statistically significant, suggesting some impact of past squared residuals on current conditional variance. The d-FIGARCH coefficients, denoted by "d-FIGARCH," represent the fractional differencing parameter in the FIGARCH model. Similar to the GARCH coefficients, all d-FIGARCH coefficients are statistically significant at the 1% level, indicating significant persistence in volatility shocks. In Panel B, diagnostic tests are conducted to assess the goodness-of-fit of the FIGARCH model. The Ljung-Box Q statistic tests the null hypothesis of no serial correlation in the squared residuals of the model. For all four financial instruments, the p-values associated with the Q statistics are greater than conventional significance levels, suggesting that the null hypothesis of no serial correlation cannot be rejected. This indicates that the FIGARCH model adequately captures the autocorrelation structure in the squared residuals. Additionally, the ARCH(10) statistics test the null hypothesis of no ARCH effects in the residuals. The p-values associated with the ARCH(10) statistics are all greater than conventional significance levels, indicating that the null hypothesis cannot be rejected. This suggests that the FIGARCH model adequately captures the ARCH effects in the residuals. Finally, the Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC) are provided as measures of model fit. Lower values of AIC and SIC indicate better model fit, and in this case, the FIGARCH model exhibits lower AIC and SIC values for all four financial instruments, suggesting good model fit.

Table 2: estimation results and diagnostic tests for the FIGARCH model

	S&P500		CAC40		NIKKEI250		EURO/USD	
Panel A: estimation results								
GARCH	0.7908*	(0.0000)	0.5726*	(0.0000)	0.6529*	(0.0000)	0.9606*	(0.0000)
ARCH	0.0009***	(0.9832)	0.0668	(0.2093)	0.1574*	(0.0099)	-0.0356	(0.3847)
d- FIGARCH	0.7820*	(0.0000)	0.5483*	(0.0000)	0.6249*	(0.0000)	0.9751*	(0.0000)
$\ln(l)$	-2767.15		-3173.86		-3347.69		-1903.66	
Panel B: diagnostic tests								
$Q^2(10)$	7.5429	(0.4793)	5.3337	(0.7213)	4.7460	(0.7843)	6.6420	(0.5756)
ARCH(10)	0.7585	(0.6692)	0.56384	(0.8444)	0.4634	0.9140	0.6416	(0.7789)
AIC	2.7515		3.0998		3.4211		1.8290	
SIC	2.7654		3.1135		3.4353		1.8426	

In Panel A of Table 3, the estimation results for the HYGARCH model are presented for four financial instruments: S&P500, CAC40, NIKKEI250, and EURO/USD. Similar to the FIGARCH model, the GARCH coefficients capture the persistence of volatility shocks in the model. All GARCH coefficients are statistically significant at the 1% level, indicating significant persistence of volatility shocks in all series. The ARCH coefficients capture the effect of past squared residuals on current conditional variance. While the ARCH coefficients for S&P500 and CAC40 are not statistically significant, indicating no significant impact of past squared residuals, the ARCH coefficients for NIKKEI250 and EURO/USD are statistically significant at the 5% and 1% levels, respectively, suggesting some impact of past squared residuals on current conditional variance. The d-FIGARCH coefficients represent the fractional differencing parameter in the HYGARCH model. Similar to the GARCH coefficients, all d-FIGARCH coefficients are statistically significant at the 1% level, indicating significant persistence in volatility shocks. Additionally, the Log Alpha (HY) coefficient captures the asymmetry in volatility shocks. While the Log Alpha (HY) coefficient is statistically significant for S&P500 at the 5% level, it is not statistically significant for the other series.

Table 3: estimation results and diagnostic tests for the HYGARCH model

	S&P500		CAC40		NIKKEI250		EURO/USD	
Panel A: estimation results								
GARCH	0.7724*	(0.0000)	0.5990*	(0.0000)	0.6660*	(0.0000)	0.7420*	(0.0000)
ARCH	0.0041	(0.9210)	0.0583	(0.2391)	0.1391**	(0.0255)	0.2738*	(0.0000)
d- FIGARCH	0.7870*	(0.0000)	0.5995*	(0.0000)	0.6655*	(0.0000)	0.4437*	(0.0050)
Log Alpha (HY)	-0.0380**	(0.0116)	-0.0359	(0.1306)	-0.0247	(-1.153)	0.0344	(0.5248)
$\ln(l)$	-2761.85		-3172.78		-3347.183		-1911.15	
Panel B: diagnostic tests								
$Q^2(10)$	10.8834	(0.2083)	5.5794	(0.6942)	5.1634	(0.7399)	9.8637	(0.2747)
ARCH(10)	1.1174	(0.3448)	0.5944	(0.8196)	0.5049	(0.8876)	0.9870	(0.4523)
AIC	2.7472		3.0997		3.4216		1.8372	
SIC	2.7639		3.1161		3.4386		1.8534	

In Panel B, diagnostic tests are conducted to assess the goodness-of-fit of the HYGARCH model. The Ljung-Box Q statistic tests the null hypothesis of no serial correlation in the squared residuals of the model. The p-values associated with the Q statistics are all greater than conventional significance levels, suggesting that the null hypothesis of no serial correlation cannot be rejected. This indicates that the HYGARCH model adequately captures the autocorrelation structure in the squared residuals. Additionally, the ARCH(10) statistics test the null hypothesis of no ARCH effects in the residuals. The p-values associated with the ARCH(10) statistics are all greater than conventional significance levels, indicating that the null hypothesis cannot be rejected. This suggests that the HYGARCH model adequately captures the ARCH effects in the residuals. Finally, the Akaike Information Criterion (AIC) and Schwarz Information Criterion

(SIC) are provided as measures of model fit. Lower values of AIC and SIC indicate better model fit, and in this case, the HYGARCH model exhibits lower AIC and SIC values for all four financial instruments, suggesting good model fit.

In Panel A of Table 4, the estimation results for the FIAPARCH model are provided for four financial instruments: S&P500, CAC40, NIKKEI250, and EURO/USD. The GARCH coefficients capture the persistence of volatility shocks in the model. All GARCH coefficients are statistically significant at the 1% level, indicating significant persistence of volatility shocks in all series. The ARCH coefficients represent the impact of past squared residuals on current conditional variance. While the ARCH coefficients for S&P500 and NIKKEI250 are statistically significant at the 1% level, indicating a significant impact of past squared residuals, the ARCH coefficients for CAC40 and EURO/USD are not statistically significant, suggesting no significant impact. The d-FIGARCH coefficients represent the fractional differencing parameter in the FIAPARCH model. All d-FIGARCH coefficients are statistically significant at the 1% level, indicating significant persistence in volatility shocks. Additionally, the Log Alpha (HY) coefficients capture the asymmetry in volatility shocks. While the Log Alpha (HY) coefficients are statistically significant for all series except EURO/USD, they vary in significance levels across the different series.

In Panel B, diagnostic tests are conducted to assess the goodness-of-fit of the FIAPARCH model. The Ljung-Box Q statistic tests the null hypothesis of no serial correlation in the squared residuals of the model. The p-values associated with the Q statistics are all significant, suggesting that the null hypothesis of no serial correlation can be rejected. This indicates that the FIAPARCH model may not adequately capture the autocorrelation structure in the squared residuals. Additionally, the ARCH(10) statistics test the null hypothesis of no ARCH effects in the residuals. The p-values associated with the ARCH(10) statistics are significant for S&P500 and CAC40, indicating that the null hypothesis can be rejected, suggesting the presence of ARCH effects in the residuals. However, for NIKKEI250 and EURO/USD, the ARCH(10) statistics are not significant, suggesting no evidence of ARCH effects. Finally, the Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC) are provided as measures of model fit. The FIAPARCH model exhibits lower AIC and SIC values for all four financial instruments, suggesting a good model fit.

Table 4: estimation results and diagnostic tests for the FIAPARCH model

	S&P500		CAC40		NIKKEI250		EURO/USD	
Panel A: estimation results								
GARCH	0.5318*	(0.0000)	0.8125*	(0.0000)	0.564269*	(0.0000)	0.7040*	(0.0000)
ARCH	0.1924*	(0.0000)	0.0782	(0.1357)	0.265628*	(0.0000)	0.2771*	(0.0000)
d-FIGARCH	0.3708*	(0.0000)	0.4188*	(0.0000)	0.4248*	(0.0000)	0.4203*	(0.0025)
γ	0.9947*	(0.0000)	0.8561*	(0.000)	0.7145*	(0.0000)	0.0846	(0.2362)
δ	1.2131 *	(0.0000)	1.2332*	(0.000)	1.1742*	(0.0000)	2.1574*	(0.0000)
$\ln(l)$	-2736.86		-3350.99		-3314.11		-1910.91	
Panel B: diagnosic tests								
$Q^2(10)$	19.5293**	(0.0122)	20.5001*	(0.0086)	12.0770	(0.1478)	10.3023	(0.2444)
ARCH(10)	2.0997**	(0.0216)	2.1147**	(0.0206)	1.2610	(0.2472)	1.0330	(0.4125)
AIC	2.723431		3.039263		3.388891		1.837959	
SIC	2.742915		3.058465		3.106238		1.856888	

4. CONCLUSIONS

In this paper, our focus lies in estimating the tail distribution for financial time series, with a particular emphasis on estimating market risk measures, such as the Value-at-Risk (VaR). To achieve this objective, we employ various Fractionally Integrated (FI) GARCH models, including FIGARCH, HYGARCH, and FIAPARCH. These models are fitted to the return data using pseudo maximum likelihood estimation techniques. Once the FI GARCH models are fitted to the data, we utilize the Generalized Pareto Distribution (GPD), as suggested by Extreme Value Theory (EVT), to model the tail of the innovations distribution. EVT provides a robust framework for modeling extreme events in financial markets, making it well-suited for estimating tail risk measures like VaR. By combining FI GARCH models with EVT, we aim to capture the long-range dependence and volatility clustering observed in financial time series, while also effectively modeling the extreme tail events that can lead to significant market losses. This integrated approach allows us to better understand and quantify the risk associated with extreme market movements, providing valuable insights for risk management practitioners. Our methodology involves several steps, including data preprocessing, model fitting, and estimation of tail parameters using EVT. We then compute VaR based on the fitted models and tail distributions, allowing us to assess the potential downside risk associated with different levels of confidence. Our research findings reveal several key characteristics of financial markets. Firstly, we observe asymmetry, fat-tail distributions, and volatility clustering, which are common features of financial time series data. These properties indicate that extreme events, both positive and negative, occur more frequently than would be expected under a normal distribution, highlighting the inherent risks associated with financial markets. Furthermore, our analysis uncovers strong evidence of long-range memory in financial market volatility. This suggests that past volatility levels have a persistent effect on future volatility, indicating the presence of long-term dependencies in the data. Importantly, we find that the Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) model effectively captures this long-range memory phenomenon. The FIAPARCH model offers a flexible framework for modeling volatility dynamics, allowing for asymmetry in the response of volatility to past shocks and incorporating long-memory effects. By incorporating these features, the FIAPARCH model provides a more accurate representation of the complex dynamics observed in financial market volatility compared to traditional ARCH/GARCH models.

To assess the predictive performance of our model, we employ two distinct statistical criteria: the unconditional coverage test proposed by Kupiec (1995) and the conditional coverage test introduced by Engel and Manganelli (2004). The unconditional coverage test evaluates the accuracy of the VaR estimates by comparing the proportion of exceedances (i.e., actual losses that exceed the estimated VaR) against the specified confidence level. A well-calibrated model should produce exceedances at a rate consistent with the chosen confidence level. The Kupiec test provides a formal statistical framework for assessing the reliability of VaR estimates based on this criterion. In contrast, the conditional coverage test examines the accuracy of VaR estimates under different market conditions, taking into account the conditional heteroscedasticity and time-varying volatility observed in financial markets. This test evaluates whether the VaR estimates adequately capture the dynamic nature of market risk and adjust accordingly to changing volatility regimes. The Engel and Manganelli test offers a robust methodology for assessing the performance of VaR models in various market environments. By subjecting our model to both the unconditional and conditional coverage tests, we aim to comprehensively evaluate its predictive accuracy and robustness across different market conditions. These statistical criteria provide valuable insights into the model's ability to effectively capture and quantify market risk, informing risk management decisions and enhancing the stability and resilience of financial institutions. The results of our analysis demonstrate that the Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) model outperforms traditional GARCH, FIGARCH, and HYGARCH models in predicting one-day ahead Value-at-Risk (VaR).

By incorporating long-range memory, volatility clustering, and fat tails into its framework, the FIAPARCH model provides a more accurate representation of the complex dynamics observed in financial time series data. Furthermore, when combined with the Extreme Value Theory (EVT) approach, the FIAPARCH model achieves superior performance in predicting VaRs compared to alternative models. This highlights the importance of incorporating advanced modeling techniques that capture the nuances of financial market behavior, such as long-range memory and extreme events, into risk management assessments and hedging strategies. Our findings underscore the significance of considering the full spectrum of market dynamics, including long-range dependencies and extreme tail events, when developing risk management frameworks. By adopting a comprehensive approach that accounts for these factors, financial institutions can enhance their ability to accurately quantify and manage market risk, thereby improving decision-making processes and mitigating potential losses. Overall, our research contributes to the ongoing efforts to improve risk management practices in financial markets by highlighting the effectiveness of advanced modeling techniques, such as the FIAPARCH-EVT approach, in predicting VaRs and enhancing risk assessment capabilities.

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