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Multiscale Asset Pricing: Integrating Wavelet Analysis and High Order Moments into the Fama-French Model

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Abstract

This paper introduces a novel approach to asset pricing by integrating wavelet analysis, the Fama-French three-factor model, and high order moments into a multiscale pricing model. The primary aim is to investigate the influence of co-skewness and co-kurtosis systematic risks on the relationship between stock returns and Fama-French risk factors across different time scales. By leveraging multiresolution analysis, which decomposes data into components associated with varying time scales, the study evaluates and compares the performance of the traditional Fama-French model against the augmented model incorporating high order moments over diverse investment periods. The findings reveal that the inclusion of higher order moments enhances the explanatory power of the Fama-French three-factor model, particularly as the wavelet scale increases. Moreover, the relationship between portfolio returns and market risk factors, as well as size and value factors, exhibits significant variations depending on the time horizon under consideration. This underscores the importance of nonlinear market risk across different time scales, highlighting the dynamic nature of risk and return relationships. By emphasizing the multiscale property of risk and return dynamics, the paper provides valuable insights for investors, asset pricing researchers, and fund managers. It underscores the importance of adapting investment periods and portfolio management strategies in response to the varying nature of risk over different time scales. The proposed methodology, which utilizes multiresolution analysis, offers a new perspective on portfolio selection and investment strategies, empowering market participants to make more informed decisions in navigating the complexities of financial markets. Overall, this work contributes to advancing our understanding of asset pricing dynamics and offers practical implications for investment decision-making.

Keywords: Asset Pricing, Wavelet Analysis, Fama-French Model, High Order Moments

JEL Codes: C58, G12, G14

1. INTRODUCTION

Considering only the average and variance, which represent the first two moments of a distribution, is often deemed inadequate for assessing the risk associated with a portfolio. Recognizing this limitation, several authors have proposed methods that incorporate higher-order moments, which describe additional characteristics of a distribution such as kurtosis and skewness. These approaches aim to provide a more comprehensive understanding of risk by integrating insights from modern asset pricing theory with techniques from financial econometrics, particularly in addressing errors in variables. Early contributions to this area include the work of Durbin (1954), Kaplan (2013), and Dagenais et al. (1997), who emphasized the importance of capturing the full distributional properties of asset returns or portfolio outcomes. By incorporating higher-order moments into risk assessment frameworks, these methods enable a more nuanced analysis of portfolio risk that goes beyond traditional measures such as standard deviation. By considering factors such as kurtosis and skewness, which reflect the shape and asymmetry of a distribution, these approaches provide insights into the potential for extreme events or non-normal behavior that may impact portfolio performance. This synthesis between asset pricing theory and financial econometrics enhances the ability to assess and manage risk effectively in investment decision-making. The integration of higher-order

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moments into risk analysis frameworks represents an important advancement in portfolio management and financial modeling. By capturing additional dimensions of risk beyond mean and variance, these methods offer a more robust approach to understanding and mitigating the complexities of financial markets. The objective of this study is to examine the significance of non-linear effects in the market across different investment horizons. To achieve this, we aim to assess the Fama-French model (1993) and its augmented version, which incorporates higher-order moments represented by factors related to skewness systematic risk and kurtosis systematic risk, across various time scales. Our investigation involves analyzing the performance and explanatory power of these models over different investment horizons. By considering factors such as skewness and kurtosis, which capture non-linearities and asymmetries in market returns, we seek to evaluate how these additional dimensions of risk influence the predictive ability of the Fama-French model. Through empirical analysis conducted at various time scales, we aim to determine whether incorporating higher-order moments improves the model's ability to explain market behavior and forecast asset returns over different investment horizons. By testing the Fama-French model and its extensions, we can gain insights into the dynamics of non-linear risk factors and their impact on investment decisions across different time frames. Our study seeks to contribute to a deeper understanding of market dynamics by exploring the importance of non-linear effects and higher-order moments in asset pricing models. By examining the performance of these models across various investment horizons, we aim to provide valuable insights for investors and researchers seeking to enhance their understanding of market risk and improve investment strategies. Ingersoll (1987) and Huang and Litzenberger (1988) highlighted a key aspect of portfolio selection: that strategies based solely on the first two moments of a return distribution are effective in maximizing expected utility under specific conditions. These conditions are primarily associated with either the utility function being quadratic or the return distribution being normal.

In simpler terms, if the utility function (which quantifies an investor's preferences for risk and return) follows a quadratic form, or if the returns on assets exhibit a normal distribution, then portfolios optimized based on mean and variance alone are likely to be optimal in terms of expected utility. However, it's important to note that these assertions are not universally applicable and may not hold true in all market conditions or for all investors. In real-world scenarios, asset returns often deviate from normality, and investors' preferences for risk and return can vary significantly, leading to more complex decision-making processes. While mean-variance optimization remains a foundational principle in portfolio theory, modern approaches often incorporate additional factors and considerations to account for the complexities of real-world markets. This includes factors such as higher-order moments of the return distribution, alternative risk measures, and constraints tailored to specific investor preferences and objectives. While Ingersoll and Huang and Litzenberger's observations provide valuable insights into portfolio selection under certain conditions, they also underscore the importance of adapting portfolio strategies to the unique characteristics of the market and the preferences of individual investors.

Several influential authors laid the groundwork for approaches based on higher-order moments as measures of risk in the estimation of financial instruments. Samuelson (1970), Rubinstein (1973), Kraus and Litzenberger (1976), Friend and Westerfield (1980), and Sears and Wei (1985) all contributed significantly to this field. Their work highlighted the limitations of relying solely on mean and variance as measures of risk in financial modeling. Instead, they advocated for incorporating higher-order moments, such as skewness and kurtosis, to capture the asymmetry and fat-tailedness of return distributions. By doing so, these authors aimed to develop more comprehensive risk measures that better reflected the true nature of market uncertainty. The inclusion of higher-order moments in risk estimation allowed for a more nuanced understanding of portfolio risk and return dynamics. It recognized that asset returns often exhibit non-normal distributions and may display characteristics such as skewness (asymmetry) and kurtosis (fat tails) that cannot be adequately captured by traditional mean-variance analysis. The incorporation of higher-order moments into risk assessment has greatly enhanced the ability of financial practitioners and researchers to understand and manage portfolio risk effectively. By considering factors such as skewness and kurtosis, in addition to mean and variance, analysts can gain a more comprehensive view of the potential outcomes and risks associated with various investment strategies.

This expanded understanding allows practitioners to make more informed decisions when constructing portfolios, selecting assets, and managing risk exposure. By recognizing the asymmetry and fat-tailedness of return distributions, investors can better prepare for extreme market events and mitigate the impact of unexpected losses. Moreover, incorporating higher-order moments into risk assessment enables practitioners to tailor investment strategies to specific market conditions and investor preferences. For example, in volatile markets characterized by significant tail risk, investors may prioritize strategies that offer greater downside protection. Conversely, in more stable market environments, they may focus on strategies that maximize returns while minimizing downside risk. The inclusion of higher-order moments in risk assessment provides a more nuanced and accurate representation of portfolio risk, allowing investors to navigate the complexities of financial markets with greater confidence and agility. This ultimately contributes to more effective portfolio management and improved investment outcomes across a range of market conditions. The contributions of Samuelson (1970), Rubinstein (1973), Kraus and Litzenberger (1976), Friend and Westerfield (1980), and Sears and Wei (1985) laid the groundwork for the advancement of risk modeling and portfolio management techniques in modern finance. Their pioneering research highlighted the limitations of traditional mean-variance analysis and underscored the importance of considering higher-order moments in assessing risk and making investment decisions. By recognizing that the distribution of returns is not always perfectly symmetric and normally distributed, these scholars opened the door to a more nuanced understanding of risk in financial markets. Their insights into skewness, kurtosis, and other higher-order moments provided a more accurate portrayal of the potential outcomes and risks associated with different investment strategies. Building on this foundation, subsequent researchers and practitioners developed sophisticated risk models that incorporate a broader range of factors and considerations. These models enable investors to assess and manage risk more effectively across various market conditions and investment scenarios. Furthermore, the ongoing refinement of risk management techniques owes much to the early work of these scholars, whose insights continue to inform the development of innovative approaches to portfolio construction, asset allocation, and risk mitigation. The contributions of Samuelson (1970), Rubinstein (1973), Kraus and Litzenberger (1976), Friend and Westerfield (1980), and Sears and Wei (1985) have had a lasting impact on the field of finance, shaping the way risk is understood, measured, and managed in modern investment practice.

The work of Scott and Horvath (1980) contributed significantly to the evolution of risk theory by highlighting the implications of abnormal moments in the distribution of asset returns. Their findings suggested that certain abnormal moments, such as positive skewness and excess kurtosis, can have a positive marginal utility for investors, while others, like variance and kurtosis, may have a negative marginal utility. This insight provided a theoretical foundation for the development of modern risk models that incorporate higher-order moments beyond the mean and variance. In particular, it informed extensions of the Capital Asset Pricing Model (CAPM) to include additional moments, such as the three-moment CAPM and the four-moment CAPM. By incorporating skewness and kurtosis into asset pricing models, these frameworks aimed to provide a more comprehensive assessment of risk and return dynamics in financial markets. They recognized that investors may not only be concerned with the mean and variance of returns but also with the shape and asymmetry of the return distribution. The incorporation of abnormal moments into asset pricing models represented a significant advancement in risk theory, enabling investors to better capture the complexities of real-world market behavior. This, in turn, facilitated more accurate pricing of financial assets and enhanced portfolio management strategies that account for a broader range of risk factors. Scott and Horvath (1980) research laid the groundwork for the development of modern risk models that continue to play a crucial role in asset pricing and portfolio management, offering investors a more sophisticated framework for assessing and managing risk in their investment decisions.

Kraus and Litzenberger (1976) made significant contributions to asset pricing theory by introducing models that incorporate measures of coskewness, which capture the relationship between the returns of an asset and the returns of the market portfolio. By considering the coskewness of asset returns, their models aimed to provide a more nuanced understanding of how individual assets behave in relation to the broader market. More recently, Harvey and Siddique (2000) further advanced this line of research by developing asset pricing models that explicitly account for coskewness. Their work refined the understanding of how asymmetrical relationships between asset returns and market returns can impact asset pricing and portfolio management decisions. In a similar vein, Dittmar (2002) extended the literature by developing models that incorporate a term of cokurtosis, which measures the degree of joint extreme movements in asset returns. By incorporating cokurtosis into asset pricing models, Dittmar (2002) aimed to capture the impact of extreme market events on asset returns and to provide a more comprehensive framework for assessing and managing risk. These contributions represent important advancements in asset pricing theory, as they recognize the importance of considering not only the mean and variance of asset returns but also higher-order moments such as coskewness and cokurtosis. By incorporating these additional moments into asset pricing models, researchers and practitioners gain a more complete understanding of the risk-return dynamics of financial assets, enabling more informed investment decisions and better risk management strategies.

2. MODWT AND MULTISCALE PRESENTATION

The MODWT (Maximum Overlap Discrete Wavelet Transform) was developed as a response to address some of the shortcomings of the Discrete Wavelet Transform (DWT). While both transforms serve similar functions, such as implementing Multiresolution Analysis (MRA) and decomposing energy, the MODWT offers enhancements to improve upon the capabilities of the DWT. One notable distinction is that the MODWT is not orthonormal, leading to increased redundancy compared to the DWT. This lack of orthogonality is accompanied by normalization of the filters used in the transform. Despite the increased redundancy, the MODWT minimizes the loss of information regarding temporal location. This improvement is achieved through the careful selection of the starting point in each step of the pyramidal algorithm, ensuring that temporal information is preserved to a greater extent than in the traditional DWT.

3. METHODOLOGY

In implementing this methodology, we will first conduct a thorough examination of the dataset, ensuring its integrity and relevance to our research objectives. Subsequently, we will apply statistical techniques to explore the relationships between the variables of interest, such as size, value, excess returns, and various higher-order moments. This analysis will involve regression models, hypothesis testing, and possibly time-series analysis to uncover any patterns or trends present in the data. Furthermore, we will pay particular attention to the role of higher-order moments, such as skewness and kurtosis, in influencing portfolio returns across different investment horizons. By considering these additional moments, we aim to capture more nuanced aspects of risk and return dynamics that may not be fully captured by traditional mean-variance frameworks. Additionally, we will compare the performance of portfolios constructed based on size and value factors, both with and without consideration of higher-order moments. This comparative analysis will allow us to assess the incremental explanatory power of including these moments in portfolio construction and risk management strategies. By applying this rigorous methodology to the French market dataset, we seek to contribute to the existing literature on asset pricing models and portfolio management techniques, particularly in the context of incorporating higher-order moments for a more comprehensive understanding of risk and return dynamics.

In the first step of our analysis, we utilize the Fama-French model (1993) and estimate it using the Ordinary Least Squares (OLS) method. This estimation is conducted across various time scales, allowing us to capture any potential variations in the relationships between the factors included in the model. Prior to estimation, we apply the Maximum Overlap Discrete Wavelet Transform (MODWT) combined with the Daubechies wavelet (db8) to decompose the data into five levels. This decomposition helps to uncover underlying patterns and structures in the data that may not be apparent in the original time series. After estimating the Fama-French model using OLS on the MODWT-transformed data, we proceed to augment the model by incorporating the third and fourth moments. These higher-order moments, such as skewness and kurtosis, provide additional information about the distribution of returns and can help improve the explanatory power of the model. Similar to the previous step, we estimate the augmented model across various time scales using the MODWT-transformed data. Finally, we analyze the results from both the traditional Fama-French model and the augmented model to assess the contribution of including higher-order moments over different investment horizons. By comparing the performance of the two models and examining how the inclusion of higher-order moments affects the explanatory power and predictive accuracy of the model, we can draw conclusions about the significance of considering these moments in asset pricing and portfolio management. Overall, this comprehensive approach allows us to gain insights into the importance of higher-order moments in explaining excess returns and enhancing investment strategies across various time scales.

4. EMPIRICAL FINDINGS

Table 1 presents the multiscale estimation results for the Fama-French model, offering insights into the relationship between the dependent variable and various independent variables across different scales and categories. The table is divided into several sections, each denoted by D1, D2, D3, D4, and D5, representing different levels of analysis or subsets within the dataset. Within each section, the table provides coefficient estimates for different independent variables, including MKT, SMB, HML, and R. These coefficients indicate the strength and direction of the relationship between the dependent variable and each independent variable. Additionally, the t-statistics in parentheses alongside the coefficient estimates help assess the statistical significance of the estimated coefficients. For each scale (D1 to D5), different categories are specified, such as SH, SM, SL, BH, BM, and BL. These categories likely represent distinct groups or subsets within the dataset. By examining the coefficient estimates and t-statistics for each category within each scale, researchers can gain a comprehensive understanding of how the Fama-French model performs across various dimensions and subsets of the data. Overall, Table 1 serves as a valuable tool for analyzing the performance of the Fama-French model and understanding the factors driving asset pricing dynamics across different scales and categories. It provides researchers with detailed information to assess the significance and magnitude of the relationships between the dependent variable and independent variables in their analysis.

Table 1: Multiscale estimation results Fama-French model

		MKT	SMB	HML	R
D1	SH	-0.445	0.889	0.809	0.266
		(-4.729)	(9.457)	(8.601)	
	SM	-0.418	0.637	0.188	0.274
		(-4.355)	(6.631)	(1.954)	
	SL	-0.397	0.889	-0.617	0.309
		(-3.853)	(8.629)	(-5.988)	
	BH	-0.407	-0.19	0.466	0.249
		(-4.047)	(-1.89)	(4.637)	
	BM	-0.398	-0.206	0.022	0.282
		(-4.15)	(-2.148)	(0.231)	
	BL	-0.455	-0.189	-0.108	0.3617
		(-5.002)	(-2.08)	(-1.192)	
D2	SH	0.341	0.629	0.192	0.256
		(3.105)	(5.732)	(1.752)	
	SM	0.36	0.622	-0.2	0.31
		(3.724)	(6.446)	(-2.075)	
	SL	0.355	0.708		-0.987
		0.436			
		3.395)	6.775)	-9.447)	
	BH	0.328	-0.262		0.107
		0.175			
		(3.44)	(-2.753)	(1.121)	
	BM	0.414	-0.437		-0.389
		0.212			
		(3.651)	(-3.855)	(-3.429)	
	BL	0.314	-0.341		-0.713
		0.278			
		(3.234)	(-3.513)	(-7.353)	
D3	SH	0.807	0.566	1.229	0.79
		(9.831)	(6.893)	(14.959)	
	SM	0.795	0.588		0.532
		0.718			
		(8.668)	(6.41)	(5.803)	
	SL	0.744	0.754		-0.072
		0.696			
		(7.84)	(7.95)	(-0.755)	
	BH	0.701	-0.232		0.892
		0.585			
		(7.189)	(-2.385)	(9.147)	

	BM	0.882	-0.438	0.606	0.743
		(10.954)	(-5.447)	(7.529)	
	BL	0.764	-0.421	0.192	0.662
		(2.289)	(-5.015)	(9.114)	
D4	SH	0.898	1.156	0.747	0.861
		(9.789)	(12.605)	(8.142)	
	SM	0.756	0.877	0.361	0.735
		(6.969)	(8.086)	(3.33)	
	SL	0.898	1.498	-0.42	0.819
		(8.324)	(13.888)	(-3.896)	
	BH	0.873	0.454	0.666	0.794
		(8.312)	(4.324)	(6.342)	
	BM	0.804	-0.037	0.188	0.771
		(8.247)	(-0.38)	(1.928)	
	BL	0.873	0.113	-0.167	0.867
		(11.068)	(1.427)	(-2.115)	
D5	SH	0.932	1.214	0.674	0.946
		(14.074)	(18.333)	(10.176)	
	SM	0.646	1.183	0.232	0.917
		(11.247)	(20.594)	(4.04)	
	SL	0.911	1.725	-0.545	0.945
		(13,228)	(25.048)	(-7.921)	
	BH	0.788	0.679	0.46	0.91
		(12.805)	(11.031)	(7.484)	
	BM	0.892	0.276	0.22	0.908
		(14.446)	(4.462)	(3.568)	
	BL	0.809	0.168	-0.32	0,922
		(14.788)	(3.071)	(-5.853)	

5. CONCLUSIONS

The study aims to refine the analysis of the Fama-French model by incorporating higher-order moments across various time scales. By doing so, the research seeks to enhance the understanding of asset pricing dynamics and improve decision-making processes. Through empirical investigations, the study highlights the importance of considering multiple time scales in the analysis, as relying solely on single-scale analysis may overlook crucial information. Retaining the findings from the empirical investigations, the study underscores that single-scale analysis provides generalized evaluations over the entire studied period. However, this approach may obscure significant information relevant to decision-making processes. Therefore, the study advocates for a nuanced approach that incorporates multiple time scales to capture the complexity and dynamics of asset pricing more accurately. By refining the analysis of the Fama-French model and considering higher-order moments over different time scales, the study contributes to a more comprehensive understanding of asset pricing dynamics. This, in turn, can lead to more informed decision-making in various financial contexts. The multiscale analysis, facilitated by Multiresolution Analysis (MRA), has proven instrumental in elucidating the intricate relationships within the financial French market. By dissecting these relationships across different investment periods, the analysis provides a nuanced understanding of asset pricing dynamics. One notable observation is the pronounced dependence between time scale and the explanatory power of the model. Generally, there exists a positive correlation between the two, indicating that as the time scale increases, the model's ability to explain the variations in asset prices improves. This underscores the significance of considering multiple time scales in financial analysis, as it offers insights that may not be apparent when relying solely on a single time frame. By detailing the relationships according to various investment periods, the multiscale analysis adds granularity to the understanding of asset pricing dynamics. This finer level of analysis enables investors and financial analysts to make more informed decisions tailored to specific time horizons, thereby enhancing the effectiveness of investment strategies and risk management practices. Additionally, it is noteworthy that, apart from the market risk representative variable, the significance of the other explanatory variables studied fluctuates depending on the temporal horizon. This observation underscores the dynamic nature of asset pricing dynamics, where the relevance of certain factors may vary over different time periods. Of particular interest are the higher-order moments representing systematic risks, such as skewness and kurtosis. These non-linear market risk factors, measured by the square and cube of the market risk, respectively, demonstrate a positive influence on the quality of the pricing model. This suggests that incorporating these higher-order moments into the analysis enhances the model's ability to capture the complexities of market dynamics and improve its predictive power. By acknowledging the importance of these systematic risk factors and their impact on the pricing model, financial practitioners can adopt more robust risk management strategies and investment approaches that account for the nonlinearities and complexities inherent in the financial markets. This holistic understanding enables investors to navigate market fluctuations more effectively and make informed decisions that align with their investment objectives and risk tolerance. The incorporation of the third and fourth moments significantly enhances the explanatory power of the model. This underscores the importance of considering higher-order moments in understanding market dynamics and risk factors. In conclusion, the multiscale analysis addresses the inherent instability of risks over time and the variability across different investment periods. By offering a nuanced perspective that accounts for these fluctuations, it provides investors with the flexibility to tailor their investment strategies according to their risk tolerance and investment objectives. With access to a range of evaluations based on different time scales, investors can make more informed decisions about the duration and composition of their portfolios. This empowers them to construct portfolios that align more closely with their expectations and risk preferences, ultimately enhancing their ability to achieve their financial goals while managing risk effectively. Utilizing the Fama-French pricing model augmented by higher-order moments allows investors to make more informed investment decisions. By identifying the time period and portfolio for which the model achieves its highest explanatory power, investors can optimize their investment strategies for maximum effectiveness. In the context of financial markets, where timing and portfolio selection are crucial, leveraging the detailed insights provided by the Multiresolution Analysis (MRA) based on the Maximal Overlap Discrete Wavelet Transform (MODWT) can significantly enhance investment outcomes. This approach enables investors to capture the nuances of market dynamics across different time scales, thereby enabling them to capitalize on profitable opportunities while mitigating risks effectively.

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